Analysis of everyday life objects in motion using video, tracker and geogebra to understand parametric equations

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Abstract

This document reports the results of applying a didactic se-quence. The objective was for students to comprehend the concept of parameter and parametric equations by way of interpreting daily life obje-cts in motion that were video recorded and analyzed using Tracker and GeoGebra. The research was based on Duval's Theory of Semiotic Re-presenta-tions and on the ACODESA methodology, which promoted indi-vidual and collaborative mathematics work. It identifies the visual record rep-resented in the video of the moving objects, the numerical and graphic records obtained using Tracker and the analytical records obtained through GeoGebra. The verbal and written records are derived from the activities of the didactic sequence, the clinical interview, the report of the activities and the group presentation of results. From the beginning of the experimental phase, the students were interested in the proposal of lear-ning mathematics in another way. The relationship between daily life and school mathematics is an important factor in providing motiva-tion for understanding parametric equations and the concept of param-eter in that students participate in development of activities where a dif-ferent view is proposed, one that is unlike the dominant teacher-cen-tered algorithmic method used in classrooms.

Keywords: Semiotic representations, Photography, Video, Tracker, Ge-oGebra.

Introduction

The didactic sequence (Tobón, Pimienta and García, 2010) is proposed in response to the rising interest in an alternative strategy from undergraduate and high school mathematics teachers. A strategy that would help them describe and understand the movement of everyday life objects (Figure 1), which would complement traditional methods used to teach parametric equations and intends for students to relate school mathematics to the analysis of the trajectory of an object in motion.

The didactic sequence includes a summary of previous content that supports acquisition of the new knowledge such as: trigonometric identities, graphing functions, parametric equation theory, conical equations and the least squares method for adjusting functions. The activities developed focused on video analysis (Bryan, 2004, 2005, 2006, 2010, s/f) with the use of Tracker and GeoGebra in individual and collaborative work. Everyday life objects in motion (Figure 1) on video that were analyzed are: a. a toy horse moving in a "nearly" circular trajectory; b. a toy train on a "nearly" elliptical, circular and straight trajectory; c. displacement of an air bubble in a plastic hose in several positions; a motorcyclist in motion; d. a runner in three mo-ments: i. starting from the rest and increasing its speed, ii. entering the stage at constant speed and maintaining it, iii. starting from rest, reaching the final goal and returning.

The didactic sequence was intended as a guide for the student to relate the problem situation with the parametric equations that describe the motion from a very different perspective to school mathematics (Arrieta and Díaz, 2015; Pantoja, Guerrero, Ulloa, Nesterova, 2016; Pantoja, 2020) that is useful, attractive and motivating. Students focus on these actions so as to obtain, interpret and understand parametric equations outside the classroom.
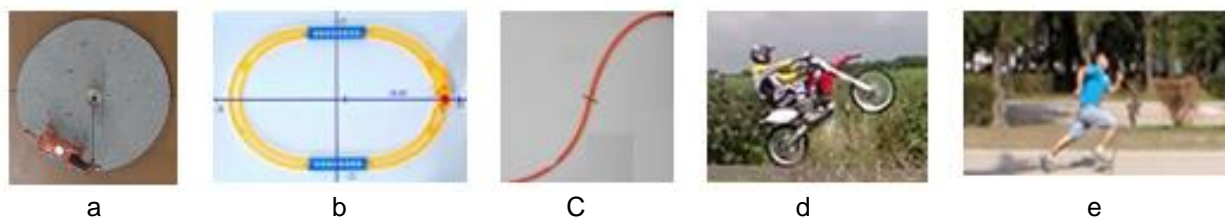

| a | b | C | d | e |

Figure 1: Everyday Life Problem Situations.

The theoretical backing for the sequence is Duval's Theory of Semiotic Representations (Duval, 1999, 2004; Duval-Sáenz, 2016). Once the video is processed with Tracker to indicate the trajectory, the numerical (a data table) and graphic (three graphs) records, which the students have to configure in order for them to be properly viewed, are generated automatically on the computer screen (Figure 2).
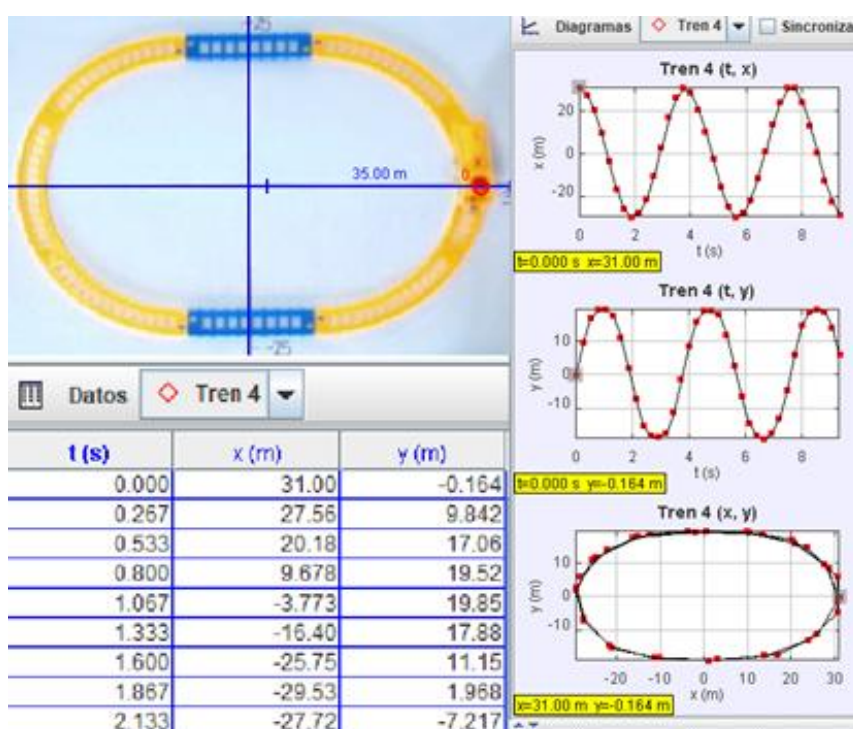

Figure 2: Visual, Graphic and Numerical Re-cords for the Train in Motion.

The analytical record of motion in parametric coordinates is $\{x = x(t), y = y(t)\}$ not automatic. It requires students to generate them in individual and collaborative work (Hitt, 2007; Hitt & Morasse, 2009), using either the limited Tracker routine or the two-variable function adjustment option in GeoGebra.

The Study

The objective of the research was to determine the effect caused by the didactic proposal on the learning of parametric equations and the understanding of the concept of parameter amongst students studying their 4th and 5th semester of industrial engineering at CUCEI. The hypothesis focuses on the idea that by using the didactic proposal students relate school mathematics to various everyday life situations, learn parametric equations and understand the concept of parameter.

Calibration of the Tracker software is done first using a ball shot through a hoop during a basketball game. Since it is considered important that the actual units of measurement of the problem situation be reflected in the analysis performed with the computer, it is important to place a unit of measurement in the recording set, which will act as interface between the real units and the units used on the computer. In this case a 2-meter ruler was located on the floor of the basketball court where the video was recorded. As background to the subject of parametric equations, we have taken the information generated by Galileo (Martínez and Guevara, 1998) on the parabolic trajectory of a steel BB when it leaves the rail on which it is in motion. In this respect Galileo writes:

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"A projectile that slides with motion composed of a uniform horizontal motion and by a naturally accelerated downward motion describes, with that motion, a semi-parabolic line." G. Galilei, *Considerations and Demonstrations..., op. cit.* (Theorem I, Proposition I) (Martinez and Guevara, 1998, p. 129).

The horizontal and uniform motion is represented by the first degree equation $x(t) = a_1 t + b_1$ and the naturally accelerated downward motion is represented by the second degree equation $y(t) = a_2 t^2 + b_2 t + c_2$, which together represent the parametric equations of a parabolic throw or freefall. From this standpoint and for the purpose of validating the work using Tracker, the video of a student shooting a ball through the basketball hoop was employed. The movement was separated and the three representative equations of the motion were determined (Figure 3). Subsequently, a comparison was made between the coefficients of the equations obtained with those that represent uniformly accelerated motion for a parabolic shot. From that an approximation of the acceleration of gravity that was considered acceptable was obtained $9.4 \frac{m}{s^2}$. This was especially the case because the initial conditions for the video recording were not ideal. For example: wind speed, given that the video was recorded outdoors, plus a lack of experience recording video, non-professional recording equipment, poor marking of the trajectory in Tracker, among others aspects.
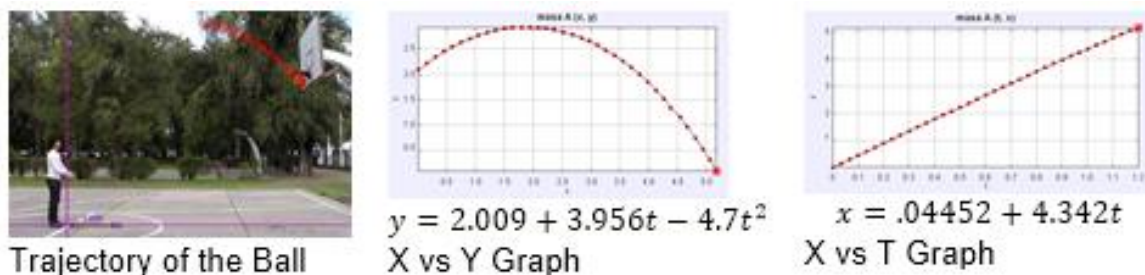

$$y = 2.009 + 3.956t - 4.7t^2$$

X vs Y Graph

$$x = .04452 + 4.342t$$

X vs T Graph

Trajectory of the Ball

Figure 3: Picture of the Shot and Graphs and Equations.

Methodology

The experimentation was developed in the workshop entitled "Using Video, Tracker and GeoGebra for Understanding Parametric Equations" with a group of 4th and 5th semester industrial engineering students at CUCEI, where a total of 30 students aged 19 to 20 years participated: 20 male and 10 female. The instruments designed for the study are:

Didactic sequence: The didactic sequence (Tobón, Pimienta and García 2010) is adapted to the selected mathematical contents so as to encourage the student to identify the parametric equations that describe motion from a very different perspective from school mathematics, which may prove useful, attractive and motivating, because the proposal places the student in a scenario where he or she focuses on obtaining and interpreting parametric equations from the environment.

Digital Video: There are 5 videos of 5 objects in motion, namely: A toy horse attached to a fixed pole, a toy train along circular, elliptical and straight paths, a bubble in a hose, a motorcyclist and a runner.

Tracker Manual: The manual describes the steps for using Tracker to perform video analysis of the bodies in motion included in the study, and to obtain numerical and graphic records of the problem situation, consisting of a data table and three graphs.

GeoGebra Handbook: The actions to be performed in order to determine the parametric equations, using adjustment of functions, from the data generated with Tracker and the instructions are: Activate the spreadsheet window, select the option of two-variable regression analysis and get the regression model.

Survey: The survey consists of 15 quantified questions using a Likert scale that evaluate four aspects: motivational, the use of technology, collaborative work and the materials used. It aims to get the opinion of students on the activities carried out in the workshop. The survey also included two questions, one open and one closed. In the first, the students were asked to express the aspects that drew their attention to the workshop; the second was aimed at selecting, from a list, the aspects that would improve the workshop: videos, GeoGebra, team integration, time, problem situations, Tracker, Worksheets and Individual Work.

Clinical Interview: The interview script consists of ten questions that were taken as a guide to the opinion of students on development of the proposal and its learning. The interview was applied to three students at the end of the experimental phase.

ACODESA methodology

The didactic sequence (Tobón, 2010), was designed for the interest of generating and proposing organized material to calculus teachers of upper and upper secondary level, to help them guide the teaching of the application of parametric equations to the description of the movement of objects in everyday life, based on the video analysis of the trajectory of the movement, with the use of selected ICT. The theoretical support included in the sequence is Duval's Theory of Semiotic Representations (2004), since once the student identifies the visual representation of the object on video, it is processed with Tracker and when pointing out the trajectory they are generated on the screen of the computer approaches numerical (one table

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of data) and graph (three graphs). The analytical expressions of motion f (x, y, k) = 0 and {x = x (t), y = y (t)} are determined in GeoGebra from the data table and the function fitting routine.

The ACODESA methodology (Hitt, 2007; Hitt and González-Martín, 2015) was included in the sequence as the central axis of the planning of activities in which it promoted individual and collaborative work, self-reflection, debate and institutionalization, as a strategy to promote learning and promote the training of students capable of posing and solving problems in different contexts of daily life. Students with their previous knowledge approximate the trajectory of the object, represented with a data table and three graphs (t, x), (t, y) and (x, y), manipulate the Tracker, locate the coordinate axes in the place more suitable for drawing the trajectory that will be analyzed in GeoGebra, for obtaining the mathematical model and with the latter obtaining the parametric equations. During this stage, first individually and later in a group, they discuss the different types of objects used and the models obtained. At the end of this stage, the instructor collects all the students' productions. In the self-reflection stage, the students individually reconstruct what was done in groups and with this knowledge is strengthened. Finally, the institutionalization process is fostered when the students present the results of the activities carried out and it is observed that the acquired knowledge coincides with the theory of parametric equations, in addition to being able to identify the parameter and its conceptualization.

Activity using Tracker and GeoGebra Software

The first activity was development of the workshop course to show how Tracker operates, together with the analysis of the video of the "almost" circular movement of the horse tied to a fixed post and the application to everyday life reflected in the grinding of the agave motored by a horse or a machine (Figure 4).

Figure 4: Toy horse and the everyday life situation of grinding agave to produce tequila.

Upon opening Tracker the main window of the program appears and then one must load the video to be analyzed. This should pose no problem for users because the software runs in the Windows environment. This eases the manipulation of Tracker routines that are used in processing the video. For instance, *Adjustment cut* is an option that allows the user to select the initial and final frames of the video, which is important as a video lasting 5 minutes at a rate of 30 frames per second would generate 9000 frames, an excessive dataset to be manipulated and the *Step Size* option should be used so as not to record an excessive number of points. In this case 290 frames are selected (Figure 5) marking one every 10 frames to generate 29 points of the path of the selected video segment. There is no rule for selecting the appropriate number of frames that show the path of an object, it is rather at the discretion of the user.

After making *the Cut Adjustment*, the axes are placed on the video as shown in Figure 5. In this case the origin is located on the fixed post. Depending on the desired analysis, the user is free to choose the position of the axes. The interface between everyday life and the virtual world is important, so in each video there is a visible mark that is taken as a reference on the computer and the *Calibration Rod* is placed there in Tracker. In this case the mark represents 10 cm.

Figure 5: Coordinated axes on the frame of the video.

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Once the video has been loaded to the computer, the object is defined as a point mass with the *Create option,* which is required to start marking the trajectory of the horse. The instructions are: *Trajectories➔New ➔Point Mass* (Figure 6). Indicating the start of the trajectory of the moving object occurs by placing the mouse on the white point of the spine of the toy horse and pressing the "*shift +click*" keys to mark the first point in the trajectory. This process is repeated 29 times as shown in the graph and data table attached to the object.
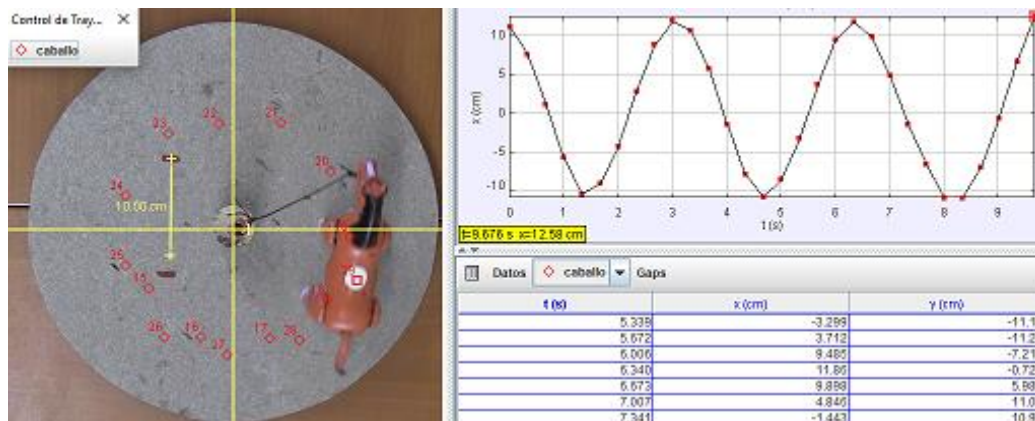

Figure 6: Contour marks on the video, graph and attached coordinate table.

A graph and a data table are generated as the trajectory is marked. Selecting the *Diagrams* option and minimizing the data table activates all three graphs at the same time (Figure 7). The user identifies the variables of the coordinate axes and changes them to x *vs t*, y *vs t*, y *vs x*, respectively.
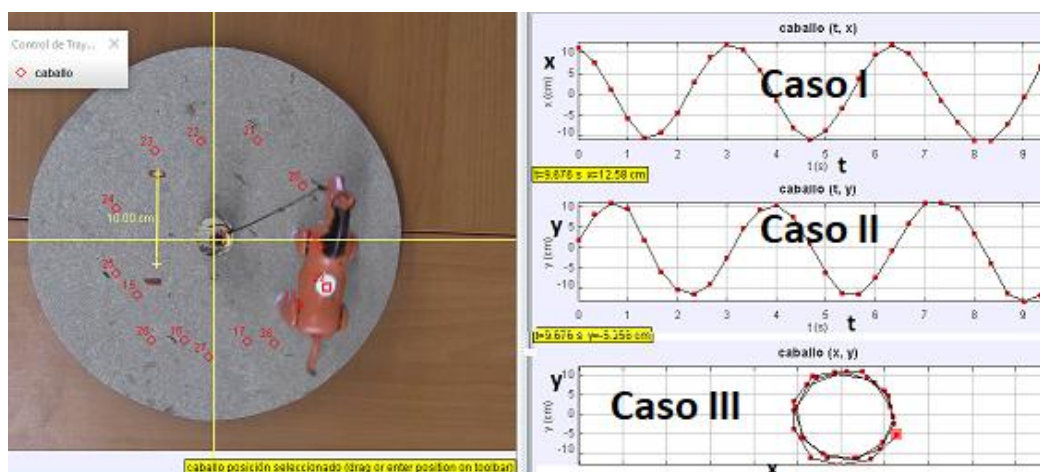

Figure 7: Three graphs on planes *x vs t*, *y vs t, y vs x.*

Conversion

The study identified several conversions between records that contributed to proper development of the learning process in students:

- Visual-Graphic: With the video of a moving object the student shifts between what he perceives and some known form, in opposite direction. That is, from the graphic to the visual, the student relates known forms with the motion of objects previously seen in the classroom.
- Visual-Numeric: Shift between what is perceived from the moving object and the coordinates of the position obtained with Tracker when performing the video analysis, using the obtained values that relate variables and time.
- Numeric-Analytical: With numerical data, students perform the adjustment to a known function using Tracker or GeoGebra routines, and then relate it to an already known function such as sinusoidal, polynomial or other.

Treatment

The treatment, linked to the theory of semiotic representations, was generated under various circumstances during development of the pilot tests and the experimental phase, as described below:

- **Analytical-analytical:** this occurs when the student rewrites the functions obtained in the analysis of the video, into a parametric form with the use of some algebraic properties and trigonometric identities. The functions adjusted with GeoGebra, in the case of the horse are $\{y = -0.68 + 20.14\,sen(1.21x + 1.83); y = -1.5 + 27.6\,sen(1.2x + 0.3)\}$, which are transformed to

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achieve an expression of the form $\{y - C_1 = \sin(ax + b); y - C_2 = \cos(ax + b)\}$, so that by squaring and adding each term the student obtains the Cartesian equation representative of the "nearly" circular trajectory of the horse.

- **Graphical-Graphical:** this occurs as relationships are established between graphs obtained from the analysis of the video of the moving object. That is, the trajectory of the horse is related to the trajectory of variables x vs t, y vs t and x vs y.

Workbook Activities

Once students attain the graphical and numeric records, they start the activities planned in the workbook. One activity asks why Tracker gives three graphs if the object has a singular motion, How can they explain this as a function of the motion? What gives rise to the graphs? The discussion of the information displayed on the computer was developed in a collaborative group aimed at emergence of the parameter from the motion of the horse, responding to the following items:

a. What do variables, constants and parameters represent in the graphs of cases I, II, III?
b. What is the relationship between the video and what is shown in the graph?

c. Explain what t means as a function of the motion.
d. Explain the relationship between the graph and the coordinates as a function of the motion.
e. Write an algebraic expression that represents the information in the graph.

Subsequently, from the data in the table calculated with Tracker, students are asked to adjust a function to the graphs shown; to relate the variables and constants of the achieved expression; and to explain the displacement of the horse mathematically. It should be noted that Tracker has a routine for that purpose, but GeoGebra has a two-variable function adjustment, which is more powerful and varied and was used to find the function for case I: $g(x) = 0.71 + 11.33\,sen(1.88x + 2.03)$.

One should not forget that GeoGebra only works with the independent variable x, a situation that the user takes into account when changing the function that has been adjusted to the variables (t, x), in which case the equation is like $g(t) = 0.71 + 11.33\,sen(1.88t + 2.03)$ -mathematical work that Duval identifies as treatment. Figure 8 shows the semiotics records of the problem situation obtained using GeoGebra: Analytical, Graphical and Numerical.

The next activity intended to encourage comparison of the expressions obtained for the parametric equations of the circle, such that students would realize their equivalence:
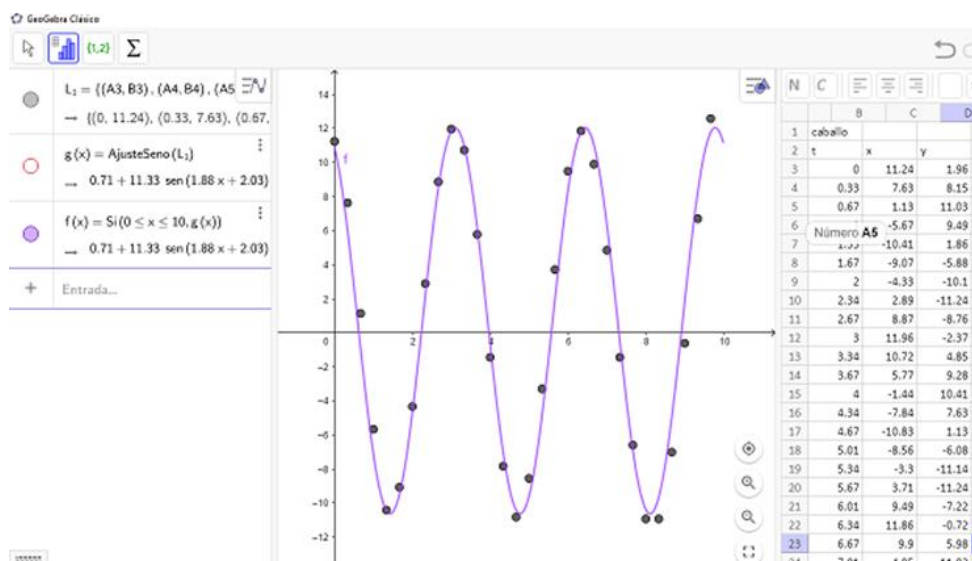
$$g(t) = 0.71 + 11.33\,sen(1.88t + 2.03) = 0.71 + 11.33\,cos\left(1.88t + 2.03 - \frac{\pi}{2}\right)$$

Figure 8: Records of semiotic representation of the motion of the horse on the (x, t) plane.

They are asked:

a. What parameters and variables can you identify in the trajectory of the horse?
b. Do you find a difference between parameter and variable? Yes (___) No (___) Which?

Video of the Motion of the Train

Once the workshop course for handling Tracker and GeoGebra was completed, each team was given one of the videos: either the train, the air bubble, the motorcycle or the runner, respectively. Students undertook the analysis of motion of the train as a collaborative activity with minimal teacher participation. It involved three "nearly" elliptical, circular and straight line trajectories (Figure 9).

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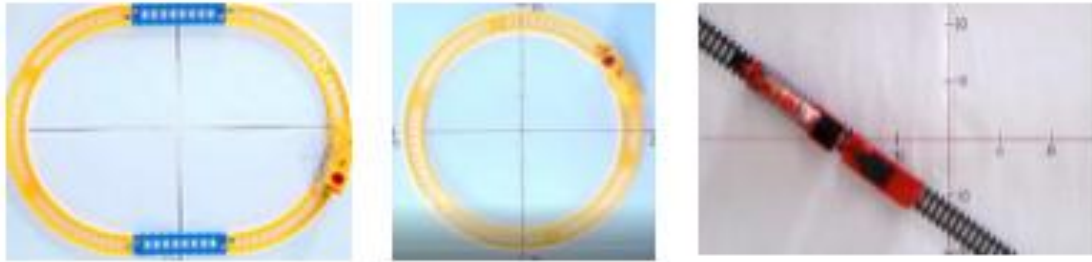
Figure 9: Nearly Circular, Elliptical or Straight Line Train Tracks.

The theory on parametric equations of the circle, ellipse and straight line was included in the workbook as a reference for development of activities with the train problem situation, as the students were able to determine the parametric equations for all three cases but still had difficulty interpreting the constants, variables and parameters.

Analysis of the Air Bubble Video

Through analysis of the videos of the air bubble inside a hose with different shapes, the activity was intended for students to be able to determine and interpret the equations that describe the motion in Cartesian and parametric forms. The hose has three positions: "nearly" vertical (Case I), "nearly" diagonal (Case II) and curvilinear form (Case III) (Figure 10).

Figure 10: Different positions of the hose with an air bubble.

Some of the questions the students answered were:

a. What type of function or equation would you associate with the shape of the trajectory described by the bubble?
b. During the trajectory:
    i. How long was the bubble active?
    ii. How far did the bubble travel when this time elapsed?
    iii. Describe how would you find the position of the bubble on the *x* axis at any given time?
    iv. Describe how you would find the position of the bubble on the *y-axis* at any given time?
c. From the table, describe the variables, parameters or constants that are involved in the movement of the object, that is, the displacement of the bubble, and describe the values that are possible.

Results

The didactic sequence (Tobón, Pimienta and García 2010) was designed for the interest of generating and proposing organized material of high school and engineering calculus teachers, which helps them guide the teaching of the application of parametric equations to the description of movement of objects of daily life, based on the video analysis of the movement's trajectory, with the use of selected ICT. The theoretical support included in the sequence is Duval's Theory of Semiotic Representations (2004), since once the student identifies the visual representation of the object on video, it is processed with Tracker and when pointing out the trajectory they are generated on the screen of the computer approaches numer-

ical (one table of data) and graph (three graphs). The analytical expressions of motion f (x, y, k) = 0 and {x = x (t), y = y (t)} are determined in GeoGebra from the data table and the function fitting routine.

Contrary to what was found in the analyzed textbooks (Lehmann, 1989; Larson, 2008; Thomas, 2010; Swokowski, 2011), the activities presented are dynamic, where the perspective of proposing an alternative strategy for their teaching, with the support of the ICT used, shows the student the route for the concept to emerge. In the analyzed literature, the subject is presented in an algorithmic way, which is meaningless and not very didactic (Lehmann. 1989, p. 265; Larson, 2008, p. 504-505), starting with a hypothetical example, which guides the student to an algorithmic process in which a point P (x, y) is used to show the trajectory of the object, while in the study, this point is

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kept in motion, as is the case of the toy horse, the "Almost" circle is visualized with its movement, in a similar way to how Galileo (Martínez and Guevara, 1998) did when he proposed the decomposition of the movement with the steel pellet moving along a rail and describing a parabolic shot. With the work carried out with the video of the moving object and with the Tracker, the emergence of the parameter concept is evidenced, since it makes sense by identifying that the parametric equations describe the coordinated movement of the dependent variables as a function of their position with respect to the time (x (t), y (t)).

The experimentation was developed in the workshop entitled "Using Video, Tracker and GeoGebra for Understanding Parametric Equations" with the students involved in the research. The workshop was held in Computer Lab 1 of the Department of Mathematics, which has 19 computers, a blackboard and a projection screen. Teams were provided with the video of the toy horse (Caballito.mp4)

showing movement on a "nearly" circular trajectory. They were asked to analyze the movement and complete the activity "Parametric Equations of the Circle". According to the results we observed that, of the 19 teams created, 11 recognized that the horse's trajectory is nearly circular, while the remaining 8 did not associate it with any form (Figure 11).

In the graphical part, eleven teams drew a circular shape with center at the origin of the coordinate axes and only 5 were able to express it algebraically. Whereas 4 related it to a sinusoidal function, but only two achieved a corresponding analytical expression. In another section they were asked to differentiate between variables, parameters and constants, and three managed to identify the angle as a parameter and two identified time, while 17 students identified speed and radius as constants.
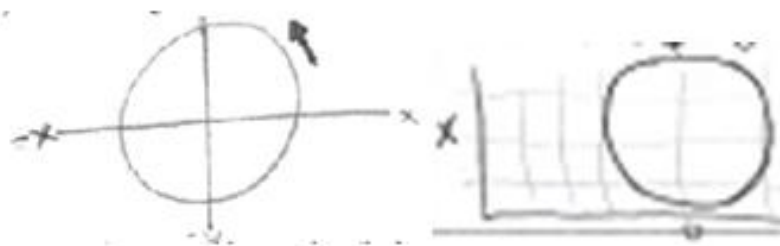

Figure 11: Results obtained by students with respect to the trajectory of the toy horse.

All teams managed to arrange the video to mark the trajectory and obtain the three graphs that are generated with Tracker and the position coordinates data table. They were asked why three graphs were included if only one object is in motion, and what each of them represents depending on the motion of the horse? They were asked about the type of function they associated with the three

graphs shown on the computer screen. Of the 19 teams, 16 managed to explain what was represented in the graph, that is, that each graph represents the movement of the object with respect to time and to either axis x or y, they also managed to find an algebraic expression for each graph (Figure 12).
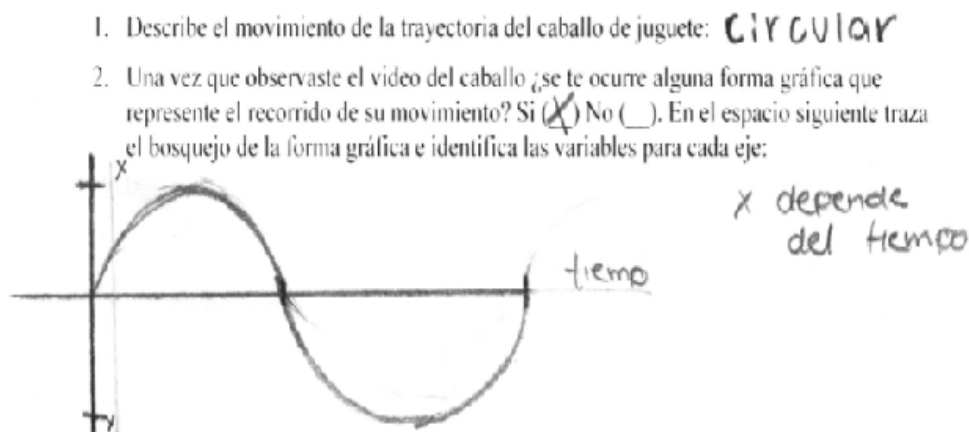

Figure 12: Graphical Representation of the Trajectory of the Object.

The students were able to adjust a trigonometric function to the graphs on planes (x, t) and (y, t),

Grafica 1: $0.21 + 9.73 \, sen (1.88t + 2.61)$

Grafica 2: $4.19 + 9.89 \, sen (1.88x + 0.41)$

and were asked about the difference between the equations that were included in the workbook $\{x(t) = Rcos(at + b), y(t) = Rsen(at + b)\}$ and those they obtained. In this regard, they managed to identify that the location of the coordinate axes on the video was not placed at the center of the pole that is attached to the horse, since the coordinates $(0.31, -0.19)$ represent the origin rather than $(0, 0)$, which is ideal for placing the coordinate axes.

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The algebraic work required to transform the equation $x(t) = 0.21 + 9.73sen(1.88t + 2.01)$ into its equivalent $x(t) = 0.21 + 9.73cos(1.88t + 0.44)$ proved challenging, since the teaching and learning of trigonometry, like Euclidean geometry and other areas of mathematics, has been minimized and work with trigonometric equations in the classroom is unusual.

Another aspect to consider in this development is that students expect the equations obtained to be identical for each group, but in the end they realize that the way of marking the points generates different equations for each group. We make this remark because students expect equations, magnitude, frequency and phase angle to be equal, yet they differ by a very small margin: $x(t) = 0.21 + 9.73sen(1.88t + 2.01) \approx x(t) = 0.21 + 9.73cos(1.88t - 2.70)$.

Session three began by making new groups of three students, who were given one of the four video options to carry out the analysis of situations for the objects in motion: the toy train, the air bubble, the motorcyclist and the runner.

For the runner the following were the questions and related answers:

Q1. Where do you think it appropriate to place the co-ordinate axes? Why?
A1. *We chose from the ground to the height of the hip of the runner as the axisso as to start from zero, it was also the center of gravity of the runner and we took the hip of the runner as our axis.*
Q2. What point did you choose to mark the trajectory? Why?
A2. *The hip of the runner since being the center of gravity it moves at a constant rate.*

The graphs shown in Figure 13 were obtained by students without using software coincided with those obtained using Tracker and were correctly explained in the presentation by the corresponding group.

Figure 13: Analysis of the case of the runner.

In general, the presentations indicated that the students had no problem performing the analysis of the three videos of the runner using Tracker and GeoGebra and they managed to correctly interpret the results obtained, since they relate the various graphs to the trajectory of the object in Figure 14.

Figure 14: Slides presented by students.

The worksheets included the question: What kind of relationship exists between school mathematics and real life? In the problem situation you analyzed, what is your choice? The answers collected gave way to the conclusion that most students agree that school mathematics coincides with real-life mathematics, an option that five of the groups selected, while the remaining three claim that school mathematics and real-life mathematics sometimes coincide.

Group Presentation

The motivation observed during the development of the activities was notorious and once their presentations were completed (Figure 15), 4th and 5th semester CUCEI industrial engineering students began writing the report on their experience learning parametric equations and understanding the concept of parameter, which was the basis for preparing the presentation and was delivered to the teacher for review. During the presentation, members of each group showed a participatory attitude, as each of them explained part of the results of their work and interpreted and explained the results obtained to the rest of their peers.
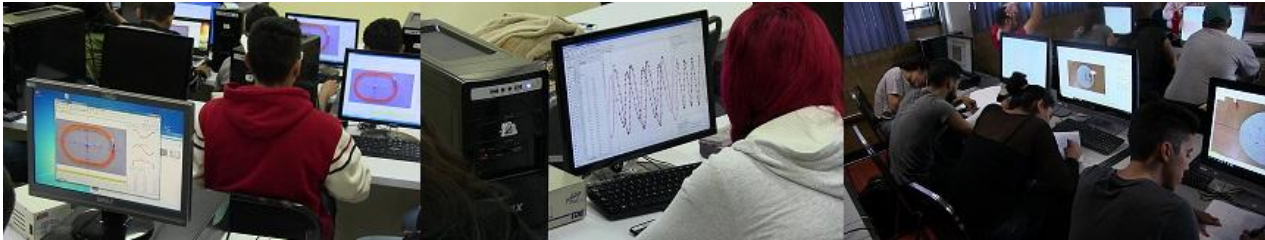

Figure 15: Students Working on the Proposed Activities.

For the group presenting the air bubble situation, the three members participated in the explanation of the data obtained (Figure 16), explained that it is important to manipulate the scale in GeoGebra in order to interpret the trajectory of the object and its representation, because when they worked on the video the graphics did not match the observations nor their prior knowledge of linear equations.
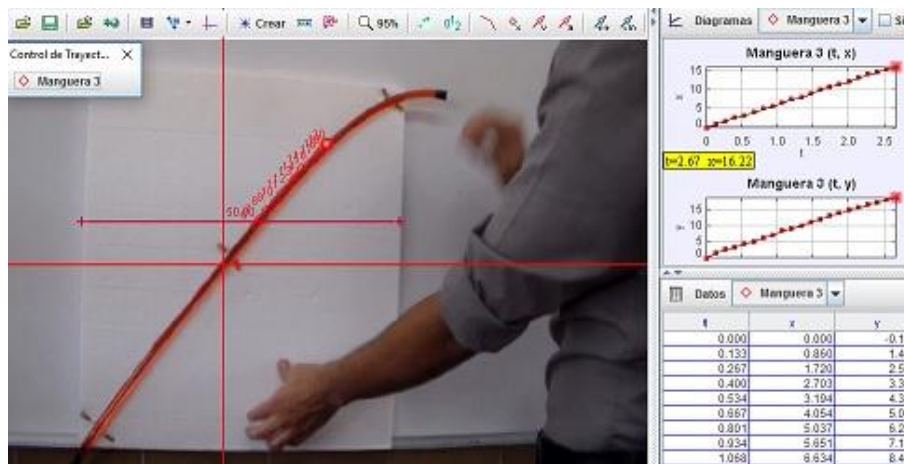

Figure 16: Group that analyzed the air bubble situation.

In the case of the motorcyclist, they explained and interpreted each of the graphs that Tracker provides in terms of the motion of the motorcyclist, and found the variables and parameters present in the said motion (Figure 17).
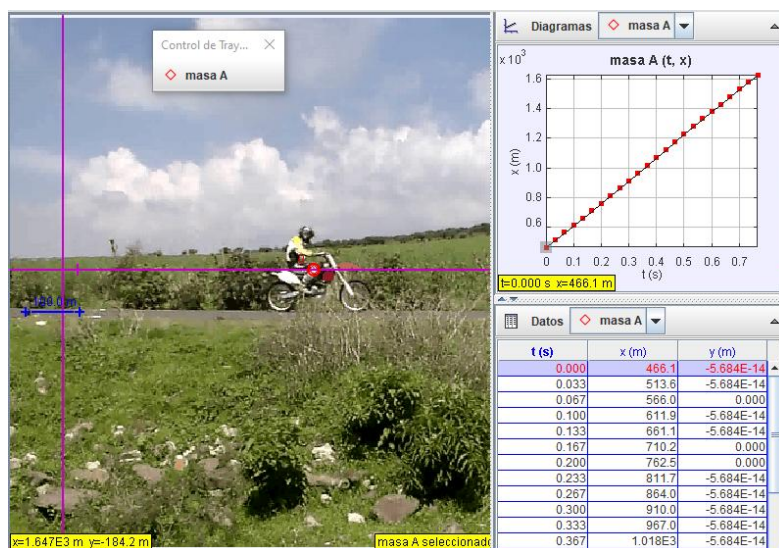

Figure 17: Team Explaining the Trajectory of the Motorcyclist.

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The group that worked with the videos of the toy train did not give their presentation due to lack of time (Figure 18), but rather used the whiteboard to explain their results, which demonstrated the clarity with which they have acquired the knowledge because they interpreted the movement of the train correctly with the semiotic representations.
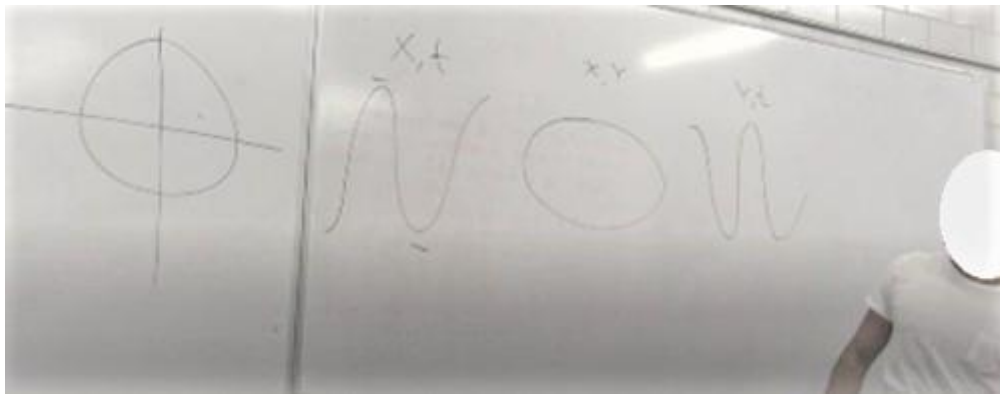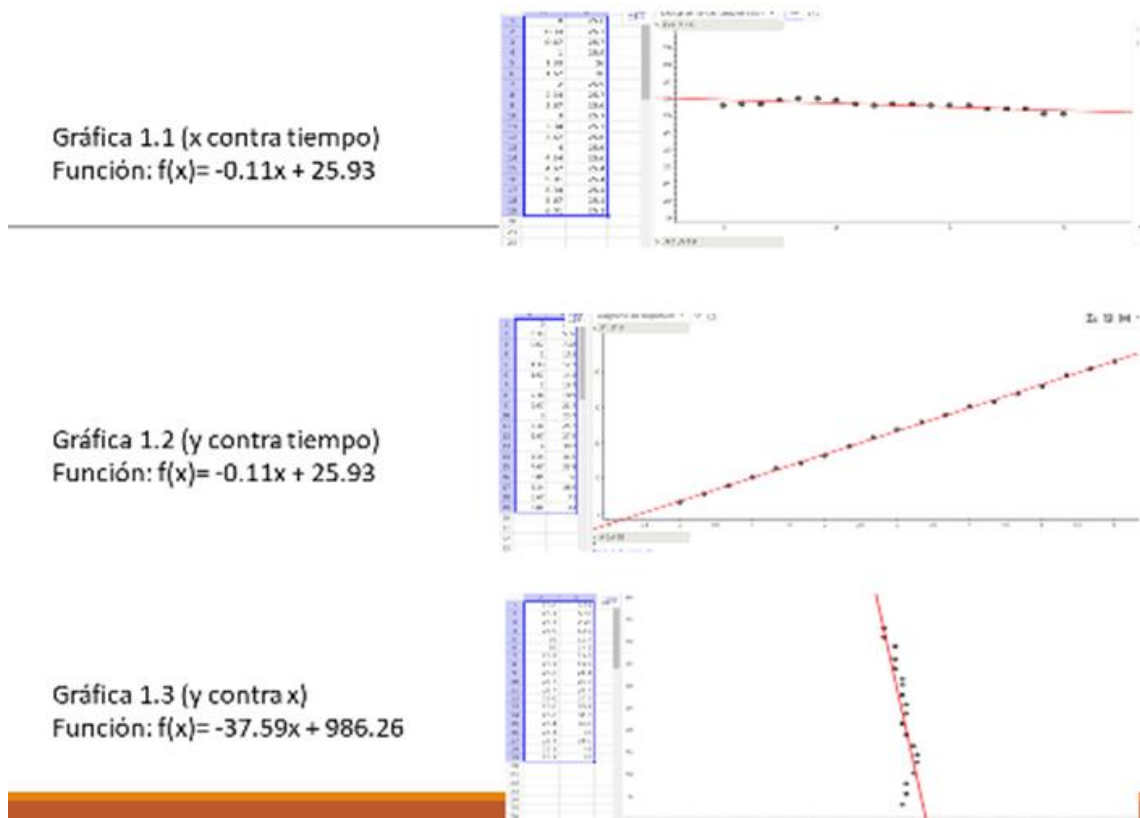

Figure 18: Semiotic representation of the Toy Train Case.

The aspects that were presented included, among others, the place where each team decided to mark the points for analysis. In general the students looked for a point in the object that would describe the trajectory: In the case of bubble, the point was on the bubble itself and they placed the origin of the coordinate axes at the start of the trajectory; for the runner, the Cartesian plane was placed at the start of the route and the trajectory was marked from the waist, because it is a point that remains "nearly" constant with respect to the ground, except when reaching the finish line and returning, which flexes in order to turn and return; for the motorcyclist they decided to use the helmet because it remained most stable; in the case of the toy train the point to be marked was clearly indicated with an easily recognizable colored circle.

During the explanation of the graphs obtained using the software, students were able to identify the relationship between the position variables (x, y) and the time variable (t) and explained that the three graphs provided by Tracker were different because each of them describes the motion of the object at a certain time, with respect to the ordinate or abscissa axis and the relationship held with the motion of the object.

In the case of the vertical motion of the air bubble, they explained that one graph remains constant because there is no translation of the object with respect to the X axis (Figure 19).

Gráfica 1.1 (x contra tiempo)
Función: f(x)= -0.11x + 25.93

Gráfica 1.2 (y contra tiempo)
Función: f(x)= -0.11x + 25.93

Gráfica 1.3 (y contra x)
Función: f(x)= -37.59x + 986.26

Figure 19: Graphs obtained by students for the motion of the air bubble.

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Another aspect that was noted during the presentations was that students identified time as a parameter, explaining that it determines the position of the object on any axis. In terms of collaborative work it was possible to create work synergies, everyone maintained participatory attitudes and when there was a need to help their fellow group members during an explanation, they did so seamlessly.

Survey

Once the presentations were completed, the students filled out an opinion survey that considered various aspects of the course, such as collaborative work, technology, the aspect of mathematical knowledge, motivation and material used during the workshop. Once the survey had been analyzed, the results show that they fully agree with this type of study, except for two items that reflect that video is not routinely used in class (70%) and that no presentations are given once the subject or course is concluded (42%), which are true.

The survey included two questions, the first was related to parameters or variables involved in the problem situation and it was based on the answers given by the students; the conclusion was that students manage to identify, but not properly classify, the intervening elements, such as distance, time and speed, that they distribute in the three sections. This denotes confusion regarding each of the concepts involved in the question and a failure to differentiate between variables, parameters and constants (Figure 20):
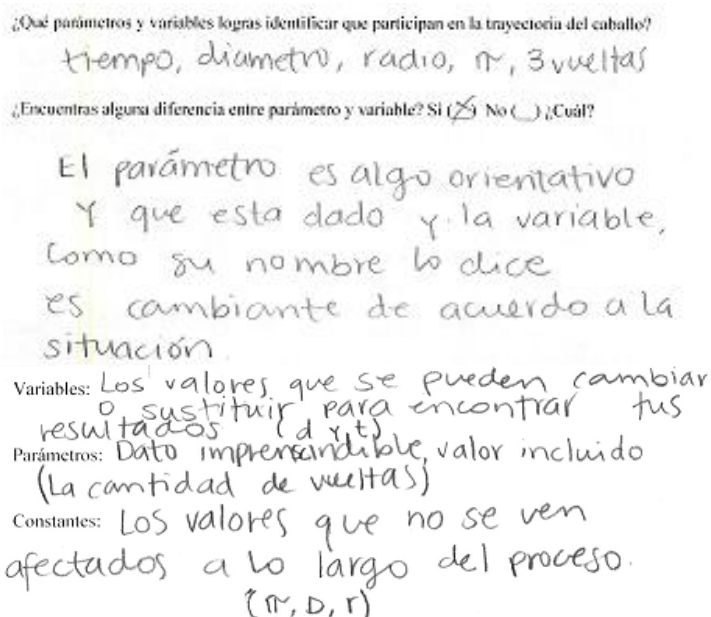

Figure 20: Comments on student submissions.

The second question asked students to identify the aspects of the workshop they found most interesting, which, after analysis, showed that they were motivated by participating in such proposals and receiving instruction in alternative form to the traditional (Figure 21).
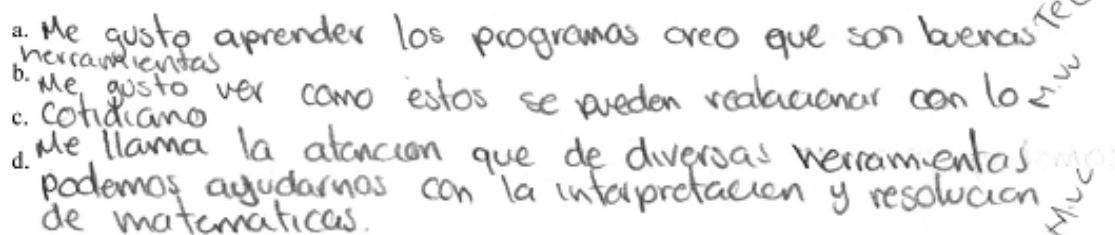

Figure 21: Aspects of the workshop that were most interesting.

In another section of the survey, students were asked to identify the aspects of the workshop that they would change. The response options are shown below, where the most influential factor was time and enjoyment when working with worksheets and technology.

Videos	6	Problem Situations	8
GeoGebra	0	Tracker	0
Group Creation	8	Worksheets	0
Time	16	Individual work	8

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From what the students expressed in the survey, we concluded that the use of problem situations to approach the topic of parametric equations and the concept of parameter, captured the interest of the students; it also reaffirms what was stated in the survey; support from Tracker and GeoGebra helped students relate mathematics to everyday life.

Interview

In an effort to better understand the impression left on the students during the development of the research, face-to-face interviews were performed with some students, they were recorded on video and their responses reaffirmed that they are motivated by this form of learning that relates school mathematics to everyday life:

- What caught your attention most about the proposal?
  - *That through the use of the two programs one can find functions in everyday life situations and that one does not know.*
  - *Using technologies and computer-based tools.*
  - *That it is not so much theory or practice, which was not as annoying as it usually is, this is more dynamic and you learn more, it is more interesting than just solving exercises.*
- What are the advantages of using Tracker and GeoGebra to find parametric equations?
  - *Well, more than anything it is easier to do and one does not get bored when doing exercises. It is more dynamic.*
  - *Quite a few, because with the use of this type of technology we realize whether we are doing things correctly or incorrectly or if the values are very close or very distant from reality.*
- What comes to your mind when you hear the word parameter and parametric equations?
  - *Interesting, at first when I didn't know anything, I thought what is that? But as knowledge is gained, one starts to understand that this is a function that describes an object's trajectory.*
- Did you find the situations used in the proposal interesting?
  - *Yes, I found it very interesting, well, I really think that in order to learn any kind of subject technology has to be included.*
  - *Yes, completely, since in classroom you see only the basics and one does not give a more accurate view of what a parametric equation can be since we only only the practical and non-theoretical approach to what it is in real life. And with these types of workshops, it is easier to learn to use technologies and also understand this topic in depth, which in this the case is parametric equations.*

According to the responses given by the students, we confirm that the use of ICT is interesting as a way to find parametric equations that describe an object in motion, in addition to the fact that these technologies allowed students to relate school mathematics to everyday life.

Conclusions

- The application of the didactic sequence for learning parametric equations {x (t), y (t)} from everyday life problem situations has a positive effect on student learning, as it contributes favorably to student motivation to carry out the activities. This proved to favor learning the topic of parametric equations and the concept of parameter.
- To provide students with a context beyond the classroom contributes to motivation and interest in mathematics. They demonstrate a participatory attitude to development of the proposal as a learning alternative. The use of real life problem situations contributes favorably and nurtures certain values in students such as punctuality, participation, respect, collaboration, initiative and tolerance, among others.
- The use of Tracker and GeoGebra contributed favorably to student learning on the subject of parametric equations and the concept of parameter.
- The collaborative work developed by students represents a favorable factor for learning, as it provides an alternative to learning individually.
- The didactic alternative offers students a context beyond school that contributes to their interest in teaching and learning mathematics.
- Prior mathematical knowledge is of paramount importance to promote learning on the subject when applying this proposal.
- In this process, the students linked the visual, graphic and numerical representations with their respective analytical expressions, which is why it is concluded that the application of Duval's theory was a motivating option to achieve that the students, from relating the different semiotic registers, understand the concept of parameter and the meaning of parametric equations from the analysis of the movement of objects in their context.

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